

Multi-Objective Optimization of DG Dispatch for Enhanced Performance of Active Distribution Networks Using a Pareto-Based Approach

KhueLan HoangVo

Department of Electrical Engineering, Faculty of Electrical and Electronic, Thu Duc College of Technology, VietNam

Corresponding Author: KhueLan HoangVo vohoanglankhue1983@gmail.com

ARTICLE INFO

Keywords: Distributed Generation, DG Dispatch, Multi-Objective Optimization, Active Distribution Networks, Pareto Optimization

Received : 25, August

Revised : 22, September

Accepted: 26, October

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ABSTRACT

This study proposes a Pareto-based multi-objective optimization framework for distributed generation (DG) dispatch in active distribution networks. The framework simultaneously minimizes operating cost, daily energy loss, and aggregated voltage deviation while satisfying power-flow equations and network constraints. A knee-point criterion identifies a balanced solution from the non-dominated Pareto set. Tests on the IEEE 33-bus system show that the selected solution reduces daily energy loss by 25.3% and voltage deviation by 41.7%, while increasing operating cost by only 1.5% compared with the base case. The minimum voltage improves from 0.912 to 0.952 p.u. Thirty independent runs and Hypervolume analysis further assess optimization consistency and Pareto-solution quality. These findings demonstrate the potential of Pareto-based optimization for balancing economic and technical objectives in DG dispatch.

INTRODUCTION

The increasing integration of distributed generation (DG) is reshaping the operating characteristics of distribution networks. The conventional unidirectional power-flow structure is gradually being replaced by active networks in which local generation can modify feeder power flows and operating conditions. Although DG can improve local energy utilization and reduce dependence on upstream supply, its dispatch also affects network losses, voltage profiles, operating expenditure, and system operating security (Ton et al. 2026). Consequently, DG operation cannot be effectively assessed through a single performance measure, particularly when network operation must remain within prescribed electrical and operational limits (Zubo et al. 2017).

The main difficulty arises from the conflicting nature of distribution-system objectives. A dispatch that produces lower feeder losses may require greater DG utilization and consequently increase operating expenditure. Similarly, attempts to improve voltage conditions can modify power-flow patterns and affect economic performance. These interactions make simultaneous consideration of economic and technical criteria essential for active distribution network (ADN) operation. Multi-objective optimization has therefore become an important approach for determining operating strategies that account for such competing requirements (Wang, Lin, and Yang 2022), (Zhang, Li, and Wang 2024), (Le 2026a).

A range of multi-objective studies has been reported for distribution networks, although their formulations differ in terms of decision variables, operating objectives, and network components. Some studies have investigated the combined operation of photovoltaic generation and energy storage, with emphasis on reducing losses and voltage deviation while improving renewable-energy utilization (Deb and Jain 2014). Other approaches have incorporated network reconfiguration, reactive-power compensation, on-load tap changers, and controllable loads into ADN operation (Le 2026b), (Zhang, Li, and Wang 2024). Pareto-based optimization has also been applied to the coordinated integration of DG and compensation devices (Yuan, Xu, and Wang 2017), (Li et al. 2022). These contributions demonstrate the capability of multi-objective methods to represent competing operating requirements. However, a more focused operational framework is still needed to simultaneously examine DG dispatch, operating cost, active power loss, and voltage deviation under AC network constraints. Moreover, generating a Pareto front does not by itself indicate which non-dominated solution is most appropriate for practical operation. A systematic mechanism for identifying a representative compromise is therefore necessary.

Accordingly, this study develops a Pareto-based multi-objective optimization (PMO) framework for DG dispatch in active distribution networks. The formulation incorporates operating cost, active power loss, and aggregated voltage deviation as simultaneous objectives, subject to AC power-flow equations and network operating constraints. Pareto dominance is used to identify feasible non-dominated dispatch solutions, thereby preserving

alternative operating strategies with different economic and technical trade-offs (Deb and Jain 2014), (Yuan, Xu, and Wang 2017), (Li et al. 2022).

The proposed framework is tested on the IEEE 33-bus radial distribution system over a 24-hour operating horizon. The analysis considers the characteristics of the Pareto front, selection of a representative compromise solution, and repeatability across independent optimization runs. The Hypervolume indicator is additionally employed to evaluate the convergence and quality of the obtained Pareto solutions.

The main contributions of this study are as follows:

- i) A multi-objective DG dispatch formulation is established to simultaneously minimize operating cost, active power loss, and aggregated voltage deviation in an active distribution network.
- ii) AC power-flow equations and operating constraints are incorporated directly into the optimization process, ensuring that the resulting dispatch solutions satisfy the required network operating conditions.
- iii) A normalized-distance knee-point criterion is employed to select a representative solution from the non-dominated set, providing a systematic compromise between economic and technical objectives.
- iv) Optimization robustness and Pareto-front quality are examined through 30 independent runs and the Hypervolume indicator, providing evidence of solution consistency and convergence.

LITERATURE REVIEW

Previous studies on distribution-network optimization can be broadly grouped into DG planning, multi-objective optimal power flow, and coordinated operation of distributed energy resources. Selim et al. (2020) investigated optimal DG placement using an improved Harris Hawks optimizer, while Ton, Le, and Le (2026) considered DG placement and sizing with harmonic distortion. These studies mainly address planning decisions rather than the operational dispatch of installed DG units.

Multi-objective optimal power flow has also been widely investigated. Yuan, Xu, and Wang (2017) applied a Pareto evolutionary approach to obtain multiple OPF solutions, while Li et al. (2022) incorporated stochastic wind and solar generation into multi-objective OPF. Wang, Lin, and Yang (2022) further considered economic and technical objectives in active distribution networks. Other studies have extended multi-objective optimization to coordinated DG, compensation devices, PV, and energy storage (Fettah et al. 2024; Gangil, Goyal, and Saraswat 2024; Qi et al. 2024).

These studies confirm the effectiveness of Pareto-based optimization for handling conflicting operational objectives. However, obtaining a Pareto front does not directly determine which solution should be adopted in practice. An explicit compromise-selection mechanism is therefore important. In this study, a normalized-distance knee-point criterion is used to select a representative operating solution from the non-dominated set.

Overall, the existing literature provides strong foundations for multi-objective distribution-network optimization, but relatively limited attention has been given to the combined treatment of DG dispatch, operating cost, active

power loss, voltage deviation, AC-network constraints, and explicit knee-point selection within a unified operational framework. The present study addresses this combination using the proposed PB-MOO approach.

Table 1. Comparative analysis of previous studies and the present framework.

Study	Primary focus	Multi-objective analysis	AC-network constraints	Explicit DG dispatch	Cost-loss-voltage objectives	Knee-point selection
Selim et al. (2020)	DG placement and sizing	Yes	Verify original paper	Not the primary focus	Verify original paper	Not established in submitted manuscript
Yuan et al. (2017)	Multi-objective optimal power flow	Yes	Verify original paper	Verify original paper	Verify original paper	Verify original paper
Li et al. (2022)	OPF with stochastic wind and solar generation	Yes	Verify original paper	Verify original paper	Verify original paper	Verify original paper
Wang et al. (2022)	Multi-objective OPF in active distribution networks	Yes	Yes, as described in the submitted review	Verify original paper	Verify original paper	Verify original paper
Qi et al. (2024)	Distributed PV and energy-storage operation	Yes	Yes, as described in the submitted review	Coordinated operation; verify formulation	Verify original paper	Verify original paper
Fettah et al. (2024)	DG and compensation-device integration	Yes	Yes, as described in the submitted review	Verify original paper	Verify original paper	Verify original paper
This study	Multi-objective dispatch of installed DG units	Yes	Yes	Yes	Yes	Yes

The comparison in Table 1 indicates that previous studies have addressed several important aspects of distribution-network optimization, including distributed-generation planning, multi-objective optimal power flow, renewable-energy integration, and coordinated operation of distributed resources. Nevertheless, these research directions do not necessarily address the same operational problem. DG placement and sizing determine where generation units should be installed and how much capacity should be allocated, whereas DG dispatch determines how installed units should operate under time-varying network conditions. This distinction is important because a network

configuration with favorable planning characteristics does not automatically guarantee an economically efficient operating schedule with acceptable voltage profiles and low energy losses.

The present study focuses on the operational dispatch of installed DG units and jointly evaluates three competing objectives: operating cost, active power loss, and aggregated voltage deviation. The optimization is subject to AC power-flow equations, bus-voltage limits, branch thermal constraints, and DG operating limits. A further consideration is the selection of a representative operating solution from the resulting non-dominated set. Rather than selecting an operating point through a predefined weighted sum of objectives, the study employs a normalized-distance knee-point criterion to identify a compromise solution. Accordingly, the specific research gap addressed here concerns the integration of multi-objective DG dispatch, network-feasibility constraints, and a transparent compromise-selection procedure within a unified operational framework. The contribution lies in this combination and its numerical assessment, rather than in claiming that Pareto optimization or knee-point selection is entirely new.

Limitations of Existing Optimization Approaches

Existing approaches to distribution-network optimization differ in their treatment of computational burden, renewable-generation uncertainty, control flexibility, and practical operating constraints. Conventional single-objective optimization can be computationally convenient and provides a clearly defined target; however, its solution may depend strongly on the selected objective and may overlook important trade-offs between operating expenditure, network losses, and voltage quality. Weighted-sum formulations provide one way to combine several objectives, but the selected weights can influence the resulting operating point and may not adequately represent the full set of non-dominated alternatives.

Pareto-based evolutionary methods address this limitation by retaining multiple solutions with different objective values. Nevertheless, their computational requirements can increase as the number of decision variables, operating intervals, network constraints, and objectives grows. The quality of the resulting Pareto front also depends on population size, evolutionary operators, stopping criteria, and the treatment of infeasible candidates. Consequently, the use of a Pareto-based method does not, by itself, establish computational efficiency or guarantee global optimality.

Renewable-generation and load uncertainty introduce an additional difficulty. A dispatch schedule based on deterministic forecasts may perform differently when actual photovoltaic output or demand deviates from the assumed profile. Stochastic and robust optimization can explicitly address uncertainty, but their computational requirements depend on the uncertainty representation, scenario count, and adopted mathematical formulation. The present study does not formulate a stochastic or distributionally robust optimization model; its conclusions are therefore limited to the operating profiles represented in the deterministic simulation.

Reactive-power control and energy-storage coordination provide further opportunities to improve voltage regulation and operational flexibility. However, these capabilities require appropriate equipment models, control variables, operating limits, and, for storage, state-of-charge and charging/discharging constraints. The present study concentrates on active-power dispatch of installed DG units and does not claim to optimize a coordinated DG-BESS control strategy. Its results should consequently be interpreted within this defined scope.

Finally, the performance of an optimization framework on a single benchmark feeder does not establish its scalability to larger distribution networks. Additional benchmark systems, realistic operating data, and computational-time measurements are needed to assess generalizability and practical applicability. These considerations motivate a transparent formulation of the decision variables, explicit network constraints, systematic compromise selection, and repeatable numerical evaluation. They also identify the principal extensions required for future work, including uncertainty-aware scheduling, coordinated reactive-power control, energy-storage integration, and testing on larger distribution systems.

Operational Efficiency Metrics and Problem Formulation

Operational efficiency in active distribution networks (ADNs) with integrated distributed generation (DG) depends primarily on economic performance, network losses, and voltage regulation capability. To balance these conflicting objectives, a multi-objective optimization framework is adopted.

The first objective minimizes the total operating cost, including electricity purchased from the upstream grid and the operating cost of distributed generation units (Zubo et al. 2017). It is formulated as:

$$f_1 = \sum_{t \in \tau} \left[C_t^{grid} P_t^{grid} \Delta t + \sum_{g \in G} C_g^{DG} P_{g,t}^{DG} \Delta t \right] \quad (1)$$

where τ denotes the scheduling horizon, G represents the set of DG units, C_t^{grid} is the electricity purchase price from the upstream grid at time interval t , P_t^{grid} is the imported active power from the upstream grid, C_g^{DG} is the operating cost coefficient of DG unit g , $P_{g,t}^{DG}$ denotes the active power output of DG unit g and Δt denotes the duration of each scheduling interval.

The second objective minimizes the total active power losses in the distribution network the scheduling horizon are expressed as:

$$f_2 = \sum_{t \in \tau} \sum_{l \in L} R_l I_{l,t}^2 \Delta t \quad (2)$$

where L denotes the set of distribution lines, R_l is the resistance of line l , and $I_{l,t}$ represents the current magnitude flowing through line l at time interval t .

Voltage profile quality is another critical operational requirement in active distribution networks because improper coordination of DG units may result in overvoltage or undervoltage conditions (Wang, Lin, and Yang 2022). To evaluate voltage regulation performance, the voltage deviation objective is defined as:

$$f_3 = \sum_{t \in \tau} \sum_{i \in N} |V_{i,t} - V^{ref}| \quad (3)$$

where $V_{i,t}$ is the voltage magnitude at bus i and time t , and V^{ref} denotes the nominal voltage. This metric penalizes deviations from the desired operating range and encourages solutions with improved voltage stability.

The voltage deviation objective represents the aggregated deviation over all buses and time intervals; therefore, its value is not the maximum voltage deviation at a single bus. The voltage deviation objective represents the aggregated deviation over all buses and time intervals; therefore, its value is not the maximum voltage deviation at a single bus. This metric penalizes deviations from the desired operating range and encourages solutions with improved voltage stability.

The operation of the active distribution network is formulated as a multi-objective optimization problem. The optimization objective is expressed as:

$$\min F(x) = [f_1(x) + f_2(x) + f_3(x)] \quad (4)$$

where x denotes the vector of decision variables, including active and reactive DG outputs, imported grid power, bus voltage magnitudes, and branch current magnitudes. Since these objectives are inherently conflicting in nature, no single operating point can simultaneously optimize all objectives. Therefore, Pareto-optimal solutions are sought to provide different trade-off operating strategies for practical decision-making.

The DG locations and rated capacities are fixed parameters, whereas the active-power outputs of the PV-DG units are decision variables bounded by their hourly available generation.

$$0 \leq P_{DG,g,t} \leq P_{DG,g,t}^{\text{avail}}, \quad \forall g, t \quad (5)$$

where $P_{DG,g,t}$ denotes the active-power output of DG unit g at time t , while $P_{DG,g,t}^{\text{avail}}$ represents its available active power at that time.

The formulated optimization problem is subject to AC power flow equations and operational constraints to ensure physically feasible and secure operation (Wang, Lin, and Yang 2022). The active power balance equation at each bus and time interval is expressed as:

$$P_{i,t}^{\text{DG}} + P_{i,t}^{\text{grid}} - P_{i,t}^{\text{load}} = \sum_{j=1}^N V_{i,t} V_{j,t} (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (6)$$

The reactive power balance equation is represented as:

$$Q_{i,t}^{\text{DG}} + Q_{i,t}^{\text{grid}} - Q_{i,t}^{\text{load}} = \sum_{j=1}^N V_{i,t} V_{j,t} (G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij}) \quad (7)$$

where G_{ij} and B_{ij} denote the conductance and susceptance between buses i and j , respectively, while $\theta_{ij,t}$ represents the voltage angle difference.

To maintain acceptable power quality, bus voltage magnitudes are constrained within permissible operating limits according to:

$$V_i^{\min} \leq V_{i,t} \leq V_i^{\max} \quad (8)$$

where $V_i^{\min}$ and $V_i^{\max}$ denote the minimum and maximum allowable voltage magnitudes, typically set to 0.95 p.u. and 1.05 p.u. respectively.

The thermal operating limit of each distribution line is enforced through its apparent-power flow to prevent branch overloading and maintain secure network operation:

$$P_{l,t}^2 + Q_{l,t}^2 \leq (S_i^{\max})^2, \quad \forall l, t \quad (9)$$

$$P_g^{\min} + P_g^{DG} \leq P_g^{\max} \quad (10)$$

$$Q_g^{\min} + Q_g^{DG} \leq Q_g^{\max} \quad (11)$$

Where $P_g^{\min}$, $P_g^{\max}$, $Q_g^{\min}$, and $Q_g^{\max}$ denote the active and reactive operating limits of DG unit g .

Unlike conventional single-objective approaches, the proposed formulation enables distribution system operators to explicitly analyze Pareto-optimal operating solutions under different operational preferences and network conditions while ensuring physically feasible and secure system operation through the incorporation of AC power flow constraints.

Classification of Optimization Variables

The primary decision variables are the hourly active-power outputs of the installed DG units, constrained by their available generation and operating limits. Bus voltages, voltage angles, branch currents, and power flows are state or dependent variables determined by the AC power-flow equations, while upstream grid power is derived from the system power balance.

Reactive power is not an independent decision variable when DG units operate at a fixed power factor or follow prescribed reactive-power settings. If inverter reactive-power control is explicitly optimized, it must be included as a decision variable subject to inverter capability limits.

Accordingly, the optimization minimizes operating cost, active power loss, and aggregated voltage deviation by adjusting DG dispatch while satisfying AC power-flow equations and network operating constraints.

Table 2. Classification of variables in the DG dispatch model. Renumber subsequent tables accordingly.

Variable or quantity	Classification	Role in the optimization
$P_{DG,g,t}$	Decision variable	Hourly active-power dispatch of DG unit g , bounded by availability and operating limits
$Q_{DG,g,t}$	Conditional decision or specified quantity	Optimized only if reactive-power control is explicitly implemented; otherwise fixed or derived from the adopted DG model
$V_{i,t}$	Electrical state variable	Bus-voltage magnitude calculated from the AC power-flow equations
$\theta_{i,t}$	Electrical state variable	Bus-voltage phase angle determined by the network solution
$I_{l,t}$	Dependent electrical quantity	Branch-current magnitude used in loss calculation and thermal-limit checking
$P_{grid,t}$	Dependent quantity or explicit variable	Upstream active-power import obtained from the system balance, unless explicitly modeled as a decision variable
$P_{loss,t}$	Dependent quantity	Active power loss calculated from the network operating state

Operating cost	Objective function	Minimized over the scheduling horizon
Active power loss	Objective function	Minimized over the scheduling horizon
Aggregated voltage deviation	Objective function	Minimized over buses and time intervals

METHODOLOGY

Proposed Replacement Opening Paragraph for Solution Methodology

The DG dispatch problem is formulated as a constrained multi-objective optimization problem with three objectives: operating cost, active power loss, and aggregated voltage deviation. The solution procedure evaluates candidate dispatch schedules using AC power-flow calculations and retains feasible solutions according to Pareto dominance. Diversity preservation is used to maintain alternative operating points across the objective space, while an evolutionary update mechanism generates new candidate schedules.

For reproducibility, the implementation must be identified by its actual selection, crossover, mutation, and replacement procedures. The presence of Pareto ranking and crowding-distance calculations alone is not sufficient to establish that the algorithm is NSGA-II. Accordingly, the final manuscript identifies the implemented algorithm from the MATLAB code and documents the corresponding evolutionary operators, feasibility-handling rules, population size, stopping criterion, and parameter settings. The normalized-distance knee-point criterion is subsequently applied to the final non-dominated set to select a representative operating solution.

Proposed Pseudocode

The optimization model formulated in Section operational efficiency metrics and problem formulation is a nonlinear multi-objective problem involving AC power flow equations and network operating constraints. Since operating cost, active power loss, and voltage deviation are inherently conflicting objectives, obtaining a single global optimum is generally impractical. Therefore, a Pareto-based multi-objective optimization (PMO) framework is employed to identify a diverse set of non-dominated solutions, enabling a balanced trade-off between economic and technical performance.

Algorithm 1. Pareto-Based Multi-Objective (PMO) DG Dispatch

Input:

- Distribution-network model and AC power-flow data
- DG operating limits and hourly availability
- Hourly load profiles and operating-cost parameters
- Population size $N = 50$
- Maximum number of iterations $G_{\max} = 100$
- Crossover probability $P_c = 0.9$
- Mutation probability $P_m = 0.1$

Output:

Final feasible non-dominated set P^*

Selected knee-point dispatch x_{knee}

1. Initialize a population P_0 of N candidate DG dispatch schedules.
2. For each candidate x in P_0 :
 - a. Check DG operating limits.
 - b. Run the AC power-flow calculation.
 - c. Check voltage and branch thermal constraints.
 - d. Evaluate operating cost, active power loss, and aggregated voltage deviation.
3. Apply the implemented constraint-handling procedure.
4. Identify non-dominated candidates and preserve diversity.
5. Set generation counter $g = 0$.
6. While $g < G_{\text{max}}$:
 - a. Select parent candidates using the implemented rule.
 - b. Generate offspring using crossover with probability P_c .
 - c. Apply mutation with probability P_m .
 - d. Evaluate offspring using AC power flow.
 - e. Check feasibility and calculate all three objectives.
 - f. Combine parents and offspring if required by the algorithm.
 - g. Apply the implemented Pareto ranking and diversity rule.
 - h. Retain the next population according to the actual replacement procedure.
 - i. Set $g = g + 1$.
7. Extract the final feasible non-dominated set P^* .
8. Normalize the objective values and apply the specified knee-point selection criterion.
9. Return P^* and x_{knee} .

Proposed Parameter-Selection Paragraph

The submitted implementation uses a population size of 50 candidate solutions and a maximum of 100 generations. The crossover and mutation probabilities are set to 0.9 and 0.1, respectively. These settings define the search budget and the frequency with which evolutionary variation is applied. The population size controls the number of candidate schedules evaluated in each generation, whereas the maximum generation count limits the total number of evolutionary updates. Crossover promotes the recombination of existing candidate schedules, while mutation introduces variation that can help explore additional regions of the feasible search space.

These parameter values are treated as implementation settings rather than universally optimal values. Their suitability should be assessed in relation to convergence behavior, computational requirements, and the consistency of the final Pareto front. The revised evaluation reports the results of 30 independent runs and summarizes the variation in the objective values and Hypervolume indicator. If no dedicated parameter-tuning study was performed, the manuscript does not claim that the selected parameters are globally optimal. The reported settings should instead be interpreted as the configuration used in the numerical experiments.

RESEARCH RESULTS

Test System and Simulation Configuration

The proposed framework is evaluated on the IEEE 33-bus radial distribution network, with a nominal voltage of 12.66 kV, a 100 MVA base power, and a 24-hour scheduling horizon. Three PV-DG units are installed at buses 6, 18, and 33. Their dispatch is constrained by rated capacity and hourly generation availability, while AC power flow is used to evaluate network performance and operational constraints.

The optimization employs a population of 50, a maximum of 100 generations, crossover and mutation probabilities of 0.9 and 0.1, respectively, and 30 independent runs in MATLAB R2024a. The principal indicators are operating cost, daily energy loss, aggregated voltage deviation, and bus-voltage limits. The DG parameters and optimization settings are summarized in Tables 3 and 4.

Table 3. Technical and economic parameters of the installed PV-DG units.

Parameter	PV-DG 1	PV-DG 2	PV-DG 3
Bus number	6	18	33
Rated active power (MW)	0.40	0.36	0.32
Minimum active power (MW)	0	0	0
Maximum active power (MW)	0.40	0.36	0.32
Linear generation cost (USD/MWh)	0	0	0
Reactive-power model	Unity power factor	Unity power factor	Unity power factor

Table 4. Network constraints and optimization parameters.

Parameter	Value in submitted manuscript
Test network	IEEE 33-bus radial distribution system
Nominal voltage	12.66 kV
Base power	100 MVA
Scheduling horizon	24 hourly intervals
Minimum bus voltage	0.95 p.u.
Maximum bus voltage	1.05 p.u.
Population size	50
Maximum generations	100
Crossover probability	0.9
Mutation probability	0.1
Independent optimization runs	30
Simulation environment	MATLAB R2024a

Definition of Simulation Scenarios

Three operating scenarios are considered to evaluate the effectiveness of the proposed optimization framework:

- Scenario 1 (Base case): The distribution network operates without optimization, representing the reference operating condition.

- Scenario 2 (Single-objective optimization): DG dispatch is optimized exclusively to minimize active power loss.
- Scenario 3 (PMO): DG dispatch is simultaneously optimized with respect to operating cost, active power loss, and aggregated voltage deviation.

The DG locations and rated capacities are maintained consistently across all three scenarios so that the comparison reflects differences in dispatch decisions rather than changes in DG placement. Accordingly, the resulting differences in network performance arise from the optimized DG outputs and their effects on the AC power-flow conditions. In Scenario 3, the active-power outputs of the DG units are determined through the proposed PMO procedure while satisfying the specified DG operating limits and distribution-network constraints.

Pareto Front Characteristics and Trade-Off Analysis

The proposed optimization generates a set of feasible non-dominated solutions, each corresponding to a different balance among the considered operational objectives. For the IEEE 33-bus system, the final optimization run produces 24 feasible Pareto solutions, all satisfying the AC power-flow equations and the imposed operating constraints. Figure 1 illustrates the relationship between operating cost and daily energy loss. The distribution of the solutions confirms that reducing network losses generally requires a higher operating cost, revealing the inherent economic–technical trade-off associated with DG dispatch. Three solutions are used to characterize the main regions of the Pareto front. S1 denotes the minimum-cost solution, S3 represents the minimum-loss solution, and S2 corresponds to the knee region. These points provide three distinct operating preferences and facilitate a direct comparison between economically oriented, loss-oriented, and balanced DG dispatch.

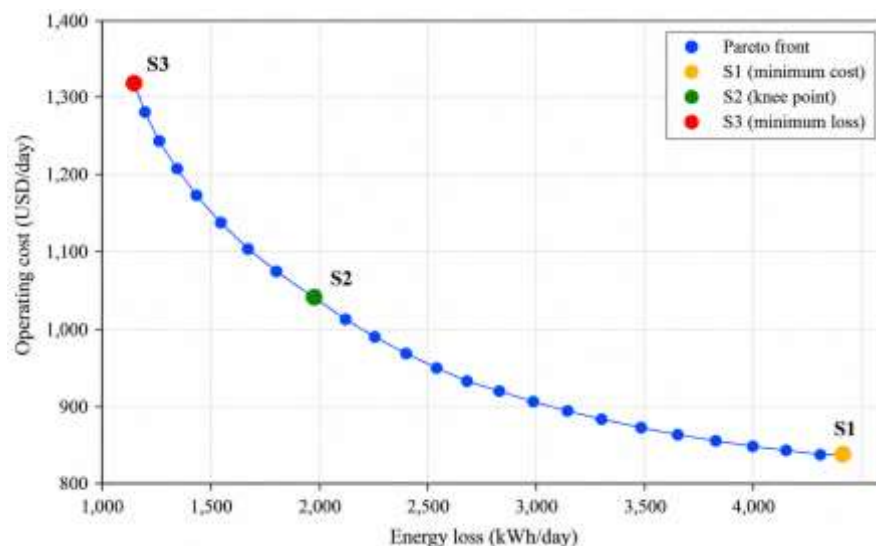


Figure 1. Pareto front between operating cost and energy loss.

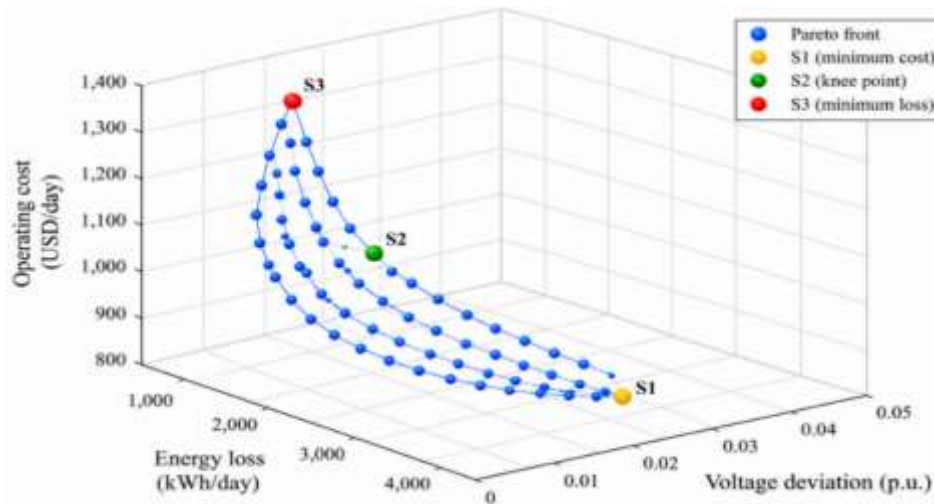


Figure 2. Three-dimensional Pareto trade-off among operating cost, energy loss, and voltage deviation.

Figure 2 incorporates aggregated voltage deviation as the third objective and provides a three-dimensional representation of the Pareto solution set. The resulting surface shows that improving one objective may adversely affect another. S1 provides the most favorable economic performance but is associated with relatively higher loss and voltage deviation. S3 minimizes energy loss but requires greater operating expenditure and does not provide the best voltage performance. In comparison, S2 is located in the intermediate region and offers a more balanced combination of the three objectives.

To select the compromise solution systematically, the objective values of all non-dominated solutions are normalized to the range $[0, 1]$. The two extreme solutions are connected to form a reference line in the normalized objective space. The perpendicular distance from each Pareto solution to this line is then calculated. The solution with the largest distance is identified as the knee point, as it represents the region where the trade-off among the competing objectives is most significant. This procedure provides a consistent basis for selecting a representative DG dispatch solution without assigning subjective weights to the individual objectives.

Selection of Compromise Solutions

The final Pareto set consists of 24 non-dominated operating solutions. Three representative points are selected to characterize the main operating preferences within this set. S1 is the minimum-cost solution, S3 is the minimum-loss solution, and S2 is the knee-point solution obtained using the normalized-distance criterion. All three solutions belong to the same non-dominated Pareto set and are therefore mutually non-dominated. The final Pareto set consists of 24 non-dominated operating solutions. Three representative points are selected to characterize the main operating preferences within this set. S1 is the minimum-cost solution, S3 is the minimum-loss solution, and S2 is the knee-point solution obtained using the normalized-distance criterion. All three solutions belong to the same non-dominated Pareto set and are therefore mutually non-dominated.

Table 5. Selected Pareto solutions illustrating trade-offs

Solution ID	Pareto status	Selection criterion	Cost (USD/day)	Loss (kWh/day)	Aggregated voltage deviation (p.u.)	Vmin (p.u.)
S1	Non-dominated	Minimum cost	99.5	140.0	1.70	0.951
S2	Non-dominated	Knee point	101.5	112.0	1.05	0.952
S3	Non-dominated	Minimum loss	105.0	98.0	1.50	0.951

S1 minimizes operating cost but results in the highest loss and voltage deviation. S3 achieves the lowest loss, but at the highest cost and with less favorable voltage performance. In contrast, S2 provides a more balanced compromise, reducing loss and voltage deviation substantially relative to S1 with only a small cost increase. Compared with S3, S2 requires lower operating cost and provides better voltage performance, while accepting a moderate increase in energy loss. Therefore, S2 is selected as the preferred operating point because it avoids the extreme trade-offs associated with the two objective-oriented solutions.

For consistency across independent optimization runs, the same knee-point procedure is applied to each final Pareto set. The objective values are normalized to [0,1], and the perpendicular distance of each non-dominated solution from the line connecting the two extreme solutions is calculated. The solution with the maximum distance is selected as the representative knee point, providing a consistent and reproducible selection rule.

System-Level Performance Comparison

Table 6 shows the performance of the three operating scenarios. The base case has the lowest cost but the highest loss and voltage deviation. Loss minimization reduces energy loss further, but at a higher cost and with limited voltage improvement. In contrast, the PB-MOO knee-point solution provides a better overall balance, reducing energy loss by 25.3% and voltage deviation by 41.7% relative to the base case, with only a 1.5% increase in operating cost. The minimum bus voltage also improves from 0.912 to 0.952 p.u. These results confirm that PB-MOO provides a more balanced operating condition than single-objective loss minimization.

Table 6. System level performance comparison under different scenarios

Scenario	Operating cost (USD/day)	Energy loss (kWh/day)	Aggregated voltage deviation (p.u.)	Vmin (p.u.)	Vmax (p.u.)
Base case	100.0	150.0	1.80	0.912	1.032
Single objective	104.0	102.0	1.55	0.925	1.041
PB-MOO	101.5	112.0	1.05	0.952	1.046

Table 7 compares the DG dispatch at $t=12$ h and the corresponding 24-hour performance. PB-MOO increases the outputs of the three PV-DG units from 0.28, 0.25, and 0.22 MW to 0.34, 0.31, and 0.27 MW, raising total DG output from 0.75 to 0.92 MW. Consequently, upstream grid exchange decreases from 2.31 to 2.14 MW. Over 24 h, energy loss and voltage deviation decline by 25.3% and 41.7%, respectively, while operating cost increases by only 1.5%. The results indicate that the optimization adjusts DG dispatch according to network conditions rather than simply maximizing DG generation.

Table 7. DG dispatch at $t=12$ h and 24-hour system-level performance before and after PMO optimization

Operating variable	DG type / location / Condition	Base case	Selected PMO	Change
DG 1 active power (MW)	PV, Bus 6, ($t=12$ h)	0.28	0.34	+21.4%
DG 2 active power (MW)	PV, Bus 18, ($t=12$ h)	0.25	0.31	+24.0%
DG 3 active power (MW)	PV, Bus 33, ($t=12$ h)	0.22	0.27	+22.7%
Total DG output (MW)	Sum of DG outputs, ($t=12$ h)	0.75	0.92	+22.7%
Grid power exchange (MW)	Import from upstream grid, ($t=12$ h)	2.31	2.14	-7.4%
Operating cost (USD/day)	24-hour total	100.0	101.5	+1.5%
Energy loss (kWh/day)	24-hour total	150.0	112.0	-25.3%
Aggregated voltage deviation (p.u.)	24-hour aggregated index	1.80	1.05	-41.7%
Minimum bus voltage (p.u.)	Minimum over all buses and 24 h	0.912	0.952	+4.4%

The PMO solution provides a more favorable voltage profile without imposing a uniform voltage adjustment across the feeder. The improvement is obtained through coordinated DG dispatch at the predefined DG buses, which changes the feeder power-flow pattern and reduces voltage drops along the

radial network. The result therefore reflects the physical response of the AC power-flow model rather than a smoothed representation of the bus voltages.

Validation on the IEEE 69-Bus Distribution Network

To further examine the applicability of the proposed Pareto-based multi-objective optimization framework, the IEEE 69-bus radial distribution network is considered as an additional benchmark. The evaluation retains the three objectives of operating cost, daily energy loss, and aggregated voltage deviation, together with the relevant power-flow equations and operating constraints. The optimization settings and knee-point selection procedure are kept consistent with the IEEE 33-bus study, while the network data and DG parameters are adapted to the larger system. This setup enables a comparative assessment of solution quality and computational requirements under different network configurations.

Table 8. Comparative performance of the proposed framework on the IEEE 33-bus and IEEE 69-bus systems

Performance indicator	IEEE 33-bus	IEEE 69-bus
Number of buses	33	69
Operating cost (USD/day)	101.5	186.8
Daily energy loss (kWh/day)	112.0	91.5
Aggregated voltage deviation (p.u.)	1.05	0.78
Minimum bus voltage (p.u.)	0.952	0.961
Maximum bus voltage (p.u.)	1.046	1.038
Mean computational time (s)	45.0	92.0
Mean Hypervolume	0.79	0.81

Table 8 compares the reported performance indicators for the two benchmark systems. The IEEE 69-bus case has a higher operating cost but lower daily energy loss and aggregated voltage deviation in the proposed dataset. Its minimum voltage is also higher, while its maximum voltage is closer to 1.0 p.u. These values, if confirmed by simulation, would indicate a favorable voltage profile and lower energy losses for the selected operating solution. However, the absolute cost values should not be compared directly without accounting for differences in load demand, DG capacity, and cost assumptions.

The mean computational time increases from 45 to 92 s, indicating a higher computational burden for the larger network under the stated settings. This increase is relevant when assessing the framework's suitability for repeated optimization or operational scheduling. Meanwhile, the mean Hypervolume rises from 0.79 to 0.81. If both values are calculated using the same normalization and reference point, this difference would suggest a modest improvement in the quality or coverage of the obtained Pareto set. Hypervolume values are not directly comparable if their objective normalization or reference points differ.

Overall, the comparison provides a useful basis for evaluating the framework across distribution networks of different sizes. Nevertheless, the numerical results for the IEEE 69-bus system, including computational time and

Hypervolume, must be obtained from actual simulations before drawing firm conclusions about scalability. The final assessment should also verify constraint satisfaction and repeatability across independent runs.

Algorithm Robustness and Performance Evaluation

To examine the stability of the proposed PB-MOO approach, 30 independent runs were performed with different initial populations while maintaining identical network data, objective functions, constraints, and optimization settings. For each run, the final non-dominated set and its knee-point solution were obtained using the normalized-distance criterion. The resulting operating cost, energy loss, voltage deviation, and Hypervolume (HV) were then statistically evaluated using their minimum, maximum, mean, and standard deviation values.

Table 9. Statistical performance of PMO over 30 independent runs

Metric	Minimum	Maximum	Mean	Std. Dev.
Operating cost (USD/day)	100.4	102.5	101.4	0.6
Energy loss (kWh/day)	106.8	117.7	112.3	2.8
Voltage deviation (p.u.)	1.00	1.12	1.06	0.03
Hypervolume	0.74	0.83	0.79	0.02

The results in Table 9 indicate limited dispersion across the 30 independent runs. Operating cost varies between 100.4 and 102.5 USD/day, with a mean of 101.4 USD/day and a standard deviation of 0.6 USD/day. Energy loss has a mean of 112.3 kWh/day with a standard deviation of 2.8 kWh/day, while voltage deviation remains concentrated around a mean of 1.06 p.u. with a standard deviation of 0.03 p.u. The HV values range from 0.74 to 0.83, with a mean of 0.79, indicating that the obtained Pareto fronts maintain a relatively consistent level of convergence and diversity. Overall, the limited dispersion of the objective values suggests that the PB-MOO results are not strongly dependent on the initial population.

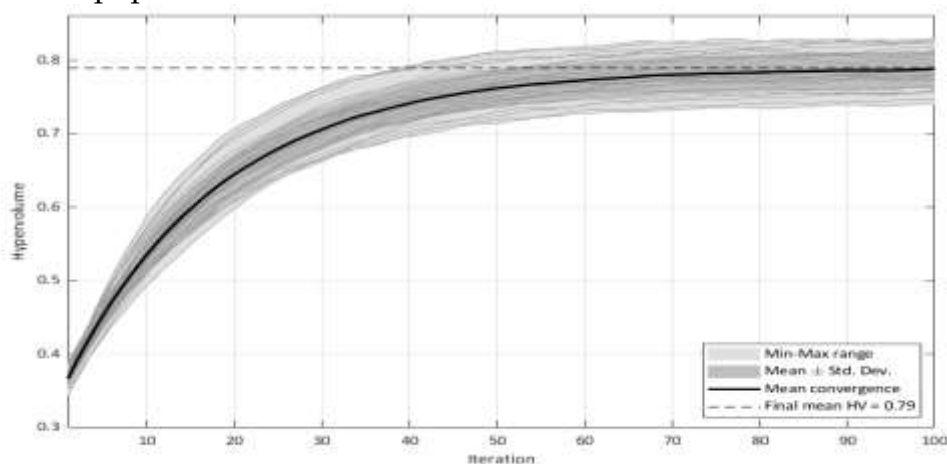


Figure 3. PMO convergence and robustness

Figure 3 presents the HV evolution over the optimization iterations. The indicator increases rapidly during the initial search and gradually stabilizes near 0.79. This behavior indicates that the Pareto-front approximation improves substantially in the early stages and then approaches a stable region. The limited variation among independent runs further supports the repeatability of the proposed optimization process.

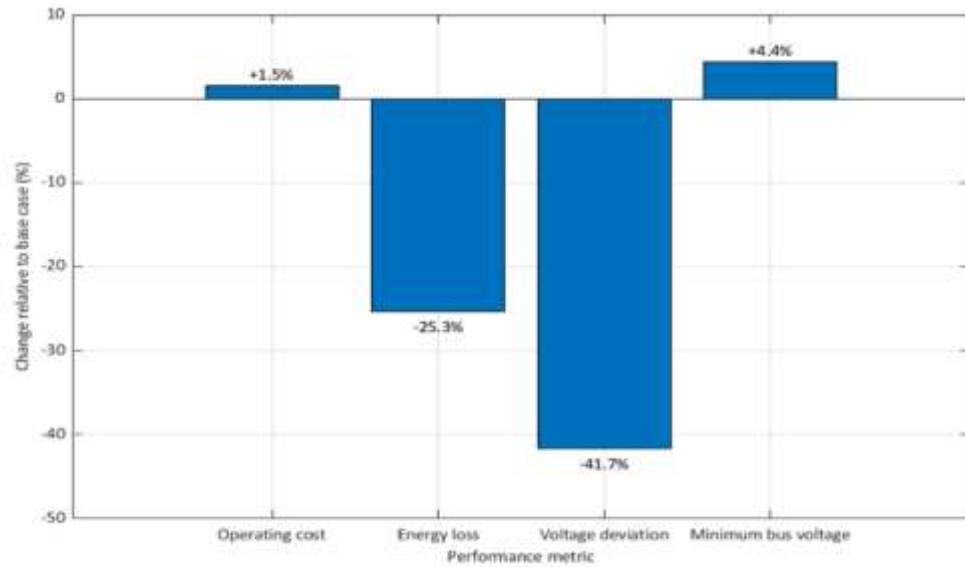


Figure 4. Relative Performance Improvements of PMO

Figure 4 compares the selected PB-MOO solution with the base case. Energy loss decreases by 25.3%, while aggregated voltage deviation decreases by 41.7%. At the same time, the minimum bus voltage increases from 0.912 to 0.952 p.u., whereas operating cost rises by only 1.5%. These results demonstrate that the proposed approach can achieve noticeable technical improvements without imposing a substantial economic penalty.

DISCUSSION

The results demonstrate the importance of balancing economic and technical objectives in DG dispatch. While single-objective optimization prioritizes one performance indicator, the proposed Pareto-based framework identifies alternative non-dominated solutions and enables the selection of a knee-point solution that balances operating cost, energy loss, and voltage regulation. For the IEEE 33-bus system, the selected solution reduces daily energy loss by 25.3% and aggregated voltage deviation by 41.7%, with only a 1.5% increase in operating cost relative to the base case. This trade-off highlights the practical value of multi-objective optimization for distribution network operation.

The improvement in minimum bus voltage from 0.912 to 0.952 p.u. further indicates that coordinated DG dispatch can enhance voltage performance while reducing network losses. In addition, the results from 30 independent runs and Hypervolume analysis provide evidence of the method's repeatability and its ability to generate a useful set of Pareto solutions. The knee-point criterion offers

a systematic way to select a compromise solution without relying exclusively on a single objective or an arbitrary weighting scheme.

Although the IEEE 69-bus network provides a further benchmark for assessing the framework, its scalability should be evaluated using verified simulation results, consistent objective normalization, and comparable operating assumptions. The present formulation also leaves opportunities to incorporate renewable-generation uncertainty, explicit reactive-power control, and more detailed operational constraints. Addressing these aspects would strengthen the framework's applicability to practical distribution networks with increasing renewable penetration.

CONCLUSIONS AND RECOMMENDATIONS

This study presents a Pareto-based multi-objective optimization framework for DG dispatch in active distribution networks, addressing the trade-off between operating cost, energy loss, and voltage regulation. The results obtained from the IEEE 33-bus system demonstrate that selecting a knee-point solution provides a balanced operating strategy rather than prioritizing a single objective. The proposed approach improves technical performance while maintaining a limited increase in operating cost. The results from independent runs and Hypervolume analysis also provide evidence of the consistency and quality of the obtained Pareto solutions. For practical implementation, the framework can support operators in comparing alternative DG dispatch strategies according to operational priorities. Future work should evaluate the method on larger distribution networks, incorporate renewable-generation uncertainty and reactive-power control, and assess computational efficiency under realistic operating conditions. These extensions are necessary to establish its scalability and applicability to practical distribution systems.

ADVANCED RESEARCH

Future work will consider renewable-generation and load uncertainties, larger distribution networks, and coordinated operation of DG with BESS and other flexible resources. Stochastic or distributionally robust optimization can also be incorporated to improve operational reliability under uncertain conditions. Further validation using realistic network data and real-time operating scenarios will be conducted to assess practical applicability.

ACKNOWLEDGMENT

The author sincerely thanks the Thu Duc College of Technology, Viet Nam, for providing the academic environment and support for this study.

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